Operations on Functions and Polynomials.

We know about ARITHMETIC operations, such as $+ \; - \; \times \; \div$

You can also perform a composition of functions as well.

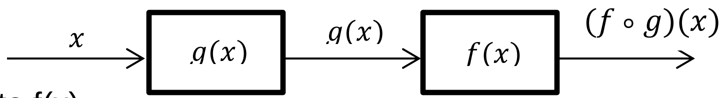
Composing functions involved “plugging one function into another one”.

Composition of functions is symbolized with an open circle: $\circ$

A composition of some function, f(x) and a function, g(x) is written: $(f \circ g)(x)$

$(f \circ g)(x)$ is read as “f of g of x” and means $f(g(x))(x)$

Graphically, this looks like:



The function g(x) is plugged into f(x).

{The range of the function g(x) is the domain of the function f(x)}

So what does this mean in real life:

Ex: $f(x) = x + 4, \;\; g(x) = 4x - 1, \;\; find \;\; (f \circ g)(x)$

$(f \circ g)(x) = f\big(g(x)\big) = so \; f(4x - 1)$ replace EVERY x in f(x) with (4x – 1)

$f(x) = x + 4$ $\qquad f(4x - 1) = (4x - 1) + 4 = 4x + 3$ $\qquad (f \circ g)(x) = 4x + 3$

Note:
$(f \circ g)(x) \neq (g \circ f)(x)$ usually.

$(g \circ f)(x) = g\big(f(x)\big) = so \; g(x + 4)$ replace EVERY x in g(x) with (x + 4)

$g(x) = 4x - 1$ $\qquad g(x + 4) = 4(x + 4) - 1 = 4x + 16 - 1$ $\qquad (g \circ f)(x) = 4x + 15$

Ex 2: $f(x) = x^2 + 1, \;\; g(x) = 2 - x, \;\; find \;\; (f \circ g)(x)$

$(f \circ g)(x) = f\big(g(x)\big) = f(2 - x) = (2 - x)^2 + 1 = (4 - 2x + x^2) + 1 = x^2 - 2x + 5$

Evaluating a composition of functions is even easier.

Ex 3: $f(x) = x^2, \;\; g(x) = 2x + 3, \;\; find \;\; (f \circ g)(1)$

$(f \circ g)(1) = f\big(g(1)\big)$ Since $g(1) = 2(1) + 3 = 5, \;\; f\big(g(1)\big) = f(5) = (5)^2 = 25$

Ex 4: $f(x) = x + 4, \;\; g(x) = 2x + 3, \;\; h(x) = 2 + x^2 \;\; find \;\; (g \circ h \circ f)(1)$

$(g \circ h \circ f)(1) = g\Big(h\big(f(1)\big)\Big)$ $\quad f(1) = 1 + 4 = 5$ $\quad h(5) = 2 + 5^2 = 27$ $\quad g(27) = 2(27) + 3 = 57$